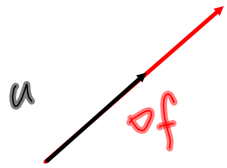
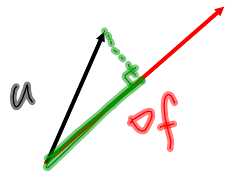
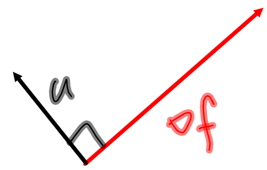


$D_u = |\nabla f|$

$= |\nabla f|\,|u|\cos\theta$

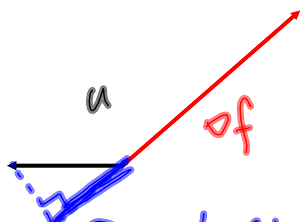


$D_u = |\nabla f|\cos\theta > 0$

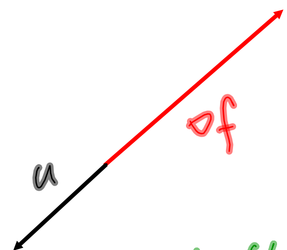


$D_u = 0$

"Level"



$D_u = |\nabla f|\cos\theta < 0$



$D_u = -|\nabla f|$

5. $f(x, y) = ye^{-x}$, $(0, 4)$, $\theta = 2\pi/3$

$f_x = -ye^{-x} = -4$

$\nabla f = \langle -4, 1 \rangle$

$f_y = e^{-x} = 1$

$-\frac{1}{2}$ $\frac{\sqrt{3}}{2}$

$D_u f = \langle -4, 1 \rangle \cdot \langle \cos\frac{2\pi}{3}, \sin\frac{2\pi}{3} \rangle$

$= 2 + \frac{\sqrt{3}}{2}$

21–26 Find the maximum rate of change of f at the given point and the direction in which it occurs.

21. $f(x, y) = y^2/x$, $(2, 4)$

$D_u f = \nabla f \cdot u = |\nabla f| \cdot \cos\theta$ max $\theta = 0$

1) $|\nabla f| \to$ max value

$f_x = -\frac{y^2}{x^2} = -4$

2) $\nabla f = k u$

$f_y = 2\frac{y}{x} = 4$

max rate $= |\langle -4, 4\rangle|$

$= 4\sqrt{2}$

Direction $= \langle -4, 4\rangle$

$= \langle -1, 1\rangle$

$u = \langle -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\rangle$

$u \to \theta$

$u = \langle \cos\theta, \sin\theta\rangle$

$\nabla f = \langle a, b\rangle$ max of D_u

max of $= a\cos\theta + b\sin\theta$

Amp $= \sqrt{a^2+b^2}$

$\sqrt{a^2+b^2}\left(\frac{a}{\sqrt{a^2+b^2}}\cos\theta + \frac{b}{\sqrt{a^2+b^2}}\sin\theta\right)$ sin α

$\sqrt{a^2+b^2}\,\sin(\theta+\alpha)$

28. Find the directions in which the directional derivative of $f(x, y) = ye^{-xy}$ at the point $(0, 2)$ has the value 1.

$f_x = -y^2 e^{-xy} = -4$

$f_y = e^{-xy} - xye^{-xy} = 1$

$\nabla f = \langle -4, 1 \rangle$

$u = \langle \cos\theta, \sin\theta \rangle$

$-4\cos\theta + \sin\theta = 1$

$\theta = \frac{\pi}{2}$